UNIVERSIDAD TECNOLÓGICA DE PANAMÁ
FACULTADES DE CIVIL e INDUSTRIL
CURSO DE MAT. III
PRUEBA PARCIAL #4 **Fecha: ____ de ___________ de 2016**

Nombre: ______________________ Cédula : ______________ N° de lista : ___ ID: A

Temas: Aplicaciones a la integral-Derivadas Parciales e Integrales Multiples

Selección Múltiple

Rellene la letra con la respuesta correcta y coloque su elección en los espacios indicados al lado de los enunciados. (no borre o tache y siga las idicaciones) Justifique cada uno de sus resultados.

____ 1 Find the first partial derivatives of the function $z = y \ln 6x$.

(F) $\dfrac{\partial z}{\partial x} = \dfrac{y}{x}, \quad \dfrac{\partial z}{\partial y} = \ln 6x$

(G) $\dfrac{\partial z}{\partial x} = \ln 6x, \quad \dfrac{\partial z}{\partial y} = \dfrac{y}{x}$

(H) $\dfrac{\partial z}{\partial x} = \dfrac{6y}{x}, \quad \dfrac{\partial z}{\partial y} = 6 \ln 6x$

(I) $\dfrac{\partial z}{\partial x} = y \ln 6x, \quad \dfrac{\partial z}{\partial y} = x \ln 6y$

(J) $\dfrac{\partial z}{\partial x} = e^{x+y}, \quad \dfrac{\partial z}{\partial y} = 0$

____ 2 Find the first partial derivatives of the function $f(x, y) = \dfrac{x - 3y}{x + 3y}$.

(A) $f_x(x, y) = \dfrac{6y}{(x + 3y)^2}, \quad f_y(x, y) = -\dfrac{6x}{(x + 3y)^2}$

(B) $f_x(x, y) = -\dfrac{6y}{(x + 3y)^2}, \quad f_y(x, y) = \dfrac{6x}{(x + 3y)^2}$

(C) $f_x(x, y) = -\dfrac{6x}{(x + 3y)^2}, \quad f_y(x, y) = \dfrac{6y}{(x + 3y)^2}$

(D) $f_x(x, y) = -\dfrac{6x}{(x - 3y)^2}, \quad f_y(x, y) = \dfrac{6y}{(x - 3y)^2}$

(E) $f_x(x, y) = \dfrac{6y}{x + 3y}, \quad f_y(x, y) = -\dfrac{6x}{x + 3y}$

____ 3 Find the first partial derivatives of the function $f(x, t) = \arctan\left(10x\sqrt{t}\right)$.

(F) $f_x(x, t) = \dfrac{10\sqrt{t}}{1 + 100x^2 t}, \quad f_t(x, t) = \dfrac{5x}{\sqrt{t}\left(1 + 100x^2 t\right)}$

(G) $f_x(x, t) = \dfrac{5x}{\sqrt{t}\left(1 + 100x^2 t\right)}, \quad f_t(x, t) = \dfrac{10\sqrt{t}}{1 + 100x^2 t}$

(H) $f_x(x, t) = \dfrac{10\sqrt{t}}{1 - 100x^2 t}, \quad f_t(x, t) = \dfrac{5x}{\sqrt{t}\left(1 - 100x^2 t\right)}$

(I) $f_x(x, t) = \dfrac{5x}{\sqrt{t}\left(1 - 100x^2 t\right)}, \quad f_t(x, t) = \dfrac{10\sqrt{t}}{1 - 100x^2 t}$

(J) $f_x(x, t) = \dfrac{5x}{\sqrt{t}\left(1 + 100x^2 t^2\right)}, \quad f_t(x, t) = \dfrac{10\sqrt{t}}{1 + 100x^2 t}$

____ 4 Find the first partial derivatives of the function $f(x, y, z) = x^4 e^{yz}$.

Ⓐ $f_x(x, y, z) = 4x^3 e^{yz}$, $f_y(x, y, z) = z x^4 e^{yz}$, $f_z(x, y, z) = y x^4 e^{yz}$

Ⓑ $f_x(x, y, z) = 4x^3 e^{yz}$, $f_y(x, y, z) = y x^4 e^{yz}$, $f_z(x, y, z) = z x^4 e^{yz}$

Ⓒ $f_x(x, y, z) = 4x^4 e^{yz}$, $f_y(x, y, z) = z x^4 e^{yz}$, $f_z(x, y, z) = y x^4 e^{yz}$

Ⓓ $f_x(x, y, z) = 4x^4 e^{yz}$, $f_y(x, y, z) = y x^4 e^{yz}$, $f_z(x, y, z) = z x^4 e^{yz}$

Ⓔ $f_x(x, y, z) = x^3 e^{yz}$, $f_y(x, y, z) = 4z x^4 e^{yz}$,
$f_z(x, y, z) = 4y x^4 e^{yz}$

____ 5 Find the first partial derivatives of the function $w = \ln(x + 4y + 5z)$.

Ⓕ $\frac{\partial w}{\partial x} = \frac{1}{x + 4y + 5z}$, $\frac{\partial w}{\partial y} = \frac{4}{x + 4y + 5z}$,
$\frac{\partial w}{\partial z} = \frac{5}{x + 4y + 5z}$

Ⓖ $\frac{\partial w}{\partial x} = \frac{4y + 5z}{x + 4y + 5z}$, $\frac{\partial w}{\partial y} = \frac{x + 5z}{x + 4y + 5z}$,
$\frac{\partial w}{\partial z} = \frac{x + 4y}{x + 4y + 5z}$

Ⓗ $\frac{\partial w}{\partial x} = \frac{1}{(x + 4y + 5z)^2}$, $\frac{\partial w}{\partial y} = \frac{4}{(x + 4y + 5z)^2}$,
$\frac{\partial w}{\partial z} = \frac{5}{(x + 4y + 5z)^2}$

Ⓘ $\frac{\partial w}{\partial x} = \frac{4y + 5z}{(x + 4y + 5z)^2}$, $\frac{\partial w}{\partial y} = \frac{x + 5z}{(x + 4y + 5z)^2}$,
$\frac{\partial w}{\partial z} = \frac{x + 4y}{(x + 4y + 5z)^2}$

Ⓙ $\frac{\partial w}{\partial x} = \frac{x}{x + 4y + 5z}$, $\frac{\partial w}{\partial y} = \frac{4y}{x + 4y + 5z}$,
$\frac{\partial w}{\partial z} = \frac{5z}{x + 4y + 5z}$

____ 6 Evaluate $\iint_D \frac{4y}{x^3 + 6}\, dA$, where $D = \{(x, y) \mid 1 \le x \le 3, 0 \le y \le 2x\}$.

Ⓐ $\frac{8}{3} \ln \frac{33}{7}$

Ⓑ $\frac{33}{7} \ln \frac{8}{3}$

Ⓒ $\frac{1}{3} \ln \frac{33}{7}$

Ⓓ $\ln \frac{33}{7}$

Ⓔ $\frac{8}{3}$

____ 7 Evaluate $\iint_D x\cos y \; dA$, where D is bounded by $y = 0$, $y = x^2$, and $x = 9$.

Ⓕ $\frac{1}{2}(1 - \cos 81)$

Ⓖ $\frac{1}{3}(1 - \cos 81)$

Ⓗ $\frac{1}{4}(1 - \sin 81)$

Ⓘ $\frac{1}{2}(1 - \sin 9)$

Ⓙ $\frac{1}{3}(1 + \cos 9)$

____ 8 Change the order of integration of $\int_0^1 \int_{14x}^{14} f(x, y)\,dy\,dx$.

Ⓐ $\int_0^{14} \int_0^{y/14} f(x, y)\,dx\,dy$

Ⓑ $\int_0^{10} \int_0^{y/10} f(x, y)\,dx\,dy$

Ⓒ $\int_0^{14} \int_0^{x/14} f(x, y)\,dy\,dx$

Ⓓ $\int_0^{10} \int_0^{x/10} f(x, y)\,dy\,dx$

Ⓔ $\int_0^{14} \int_{y/14}^{y} f(x, y)\,dx\,dy$

____ 9 Change the order of integration of $\int_1^9 \int_0^{\ln x} f(x, y)\,dy\,dx$.

Ⓕ $\int_0^{\ln 9} \int_{e^y}^{9} f(x, y)\,dx\,dy$

Ⓖ $\int_0^{\ln 9} \int_{e^y}^{e^9} f(x, y)\,dx\,dy$

Ⓗ $\int_0^{\ln 9} \int_{e^x}^{9} f(x, y)\,dy\,dx$

Ⓘ $\int_0^{\ln 9} \int_{e^x}^{e^9} f(x, y)\,dy\,dx$

Ⓙ $\int_0^{e^9} \int_{e^y}^{9} f(x, y)\,dx\,dy$

____ 10 Evaluate $\int_{0}^{1}\int_{12y}^{12} e^{x^2}\,dx\,dy$ by reversing the order of integration.

Ⓐ $\dfrac{e^{144} - 1}{24}$

Ⓑ $\dfrac{e^{144} + 1}{24}$

Ⓒ $\dfrac{e^{144}}{24}$

Ⓓ $\dfrac{e^{144} + e}{24}$

Ⓔ $\dfrac{e^{144} - e}{24}$

____ 11 Evaluate $\int_{0}^{1}\int_{\sqrt{y}}^{1} \sqrt{x^3 + 10}\,dx\,dy$ by reversing the order of integration.

Ⓕ $\frac{2}{9}\left(11^{3/2} - 10^{3/2}\right)$

Ⓖ $\frac{2}{9}\left(11^{3/2} + 10^{3/2}\right)$

Ⓗ $\frac{9}{2}\left(11^{3/2} - 10^{3/2}\right)$

Ⓘ $\frac{9}{2}\left(11^{3/2} + 10^{3/2}\right)$

Ⓙ $\frac{9}{2}\left(11^{5/2} - 10^{3/2}\right)$

____ 12 Evaluate $\int_{0}^{1}\int_{x^2}^{1} x^3 \sin\left(9y^3\right)\,dy\,dx$ by reversing the order of integration.

Ⓐ $\frac{1}{108}(1 - \cos 9)$

Ⓑ $\frac{1}{108}(1 + \cos 9)$

Ⓒ $\frac{1}{108}(1 - \sin 9)$

Ⓓ $\frac{1}{108}(1 + \sin 9)$

Ⓔ $\frac{1}{108}(\sin 9 + \cos 9)$

____ 13 Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = (x - 4)^4,\ 8x - y = 32;$ about $x = 16.$

Ⓕ $V = \pi \int_0^{16} \left\{ \left[16 - \left(\frac{1}{8} y + 4 \right) \right]^2 - \left[16 - \left(4 + \sqrt[4]{y} \right) \right]^2 \right\} dy$

Ⓖ $V = \pi \int_0^{16} \left\{ \left[16 - \left(\frac{1}{8} y + 4 \right) \right]^2 + \left[16 - \left(4 + \sqrt[4]{y} \right) \right]^2 \right\} dy$

Ⓗ $V = \pi \int_4^{6} \left\{ \left[16 - \left(\frac{1}{8} y + 4 \right) \right]^2 - \left[16 - \left(4 + \sqrt[4]{y} \right) \right]^2 \right\} dy$

Ⓘ $V = \pi \int_4^{6} \left\{ \left[16 - \left(\frac{1}{8} y + 4 \right) \right]^2 + \left[16 - \left(4 + \sqrt[4]{y} \right) \right]^2 \right\} dy$

Ⓙ $V = \pi \int_0^{4} \left\{ \left[16 - \left(\frac{1}{8} y + 4 \right) \right]^2 - \left[16 - \left(4 + \sqrt[4]{y} \right) \right]^2 \right\} dy$

____ 14 Given the curves $y = \ln(x + 1),\ y = x^{10}.$

Find the intersection points, and approximate the volume obtained by rotating the region bounded by the curves about the x axis.

Ⓐ $x_1 = 0$ and $x_2 \approx 0.96,\ V \approx 0.469$

Ⓑ $x_1 = 0.04$ and $x_2 \approx 1.06,\ V \approx 0.516$

Ⓒ $x_1 = 0.05$ and $x_2 \approx 1.15,\ V \approx 0.563$

Ⓓ $x_1 = 0$ and $x_2 \approx 0.67,\ V \approx 0.328$

Ⓔ $x_1 = 0$ and $x_2 \approx 0.58,\ V \approx 0.281$

____ 15 Given the curves $y = 2\sin\left(x^2\right),\ y = e^{\frac{x}{3}} + e^{-3x}.$

Find the intersection points, and approximate the volume obtained by rotating the region bounded by the curves about the x-axis.

Ⓕ $x_1 = 0.89$ and $x_2 \approx 1.47,\ V \approx 1.918$

Ⓖ $x_1 = 0.98$ and $x_2 \approx 1.62,\ V \approx 2.110$

Ⓗ $x_1 = 1.07$ and $x_2 \approx 1.76,\ V \approx 2.302$

Ⓘ $x_1 = 1.16$ and $x_2 \approx 1.91,\ V \approx 2.493$

Ⓙ $x_1 = 1.25$ and $x_2 \approx 2.06,\ V \approx 2.685$

Numeric Response

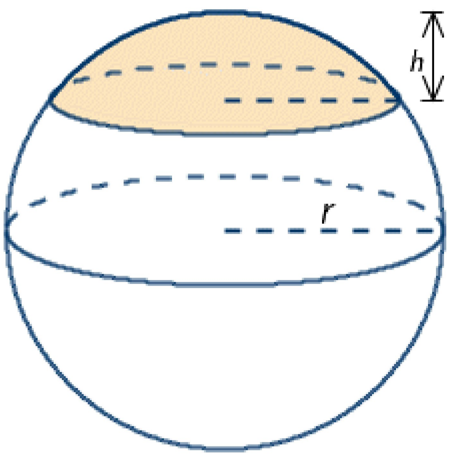
16 Find the area of the region enclosed by the curves $y = 4x$ and $y = 4x^2$, accurate to two decimal places.

17 Find the area of the region enclosed by the curves $y = 64 - x^2$ and $y = x^2 - 8$, accurate to two decimal places.

Short Answer

18 Use the arc length formula $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ to find the length of the curve $y = 7 - 3x$, $-4 \le x \le 7$. Check your answer by noting that the curve is a line segment and calculating its length by the distance formula.

19 Find the volume of a cap of a sphere with radius r and height h.



20 Find the volume of a frustum of a pyramid with square base of side b, square top of side a, and height h.