FÓRMULAS DE LAS FUNCIONES HIPERBÓLICAS

❒ Definición de funciones hiperbólicas

1. $sinh\,x = \frac{e^x - e^{-x}}{2}$
2. $cosh\,x = \frac{e^x + e^{-x}}{2}$
3. $tanh\,x = \frac{sinhx}{cosh} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
4. $csch\,x = \frac{1}{sinhx} = \frac{2}{e^x - e^{-x}}$
5. $sech\,x = \frac{1}{coshx} = \frac{2}{e^x + e^{-x}}$
6. $coth\,x = \frac{1}{tanhx} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

❒ Identidades de funciones hiperbólicas

1. $cosh^2 x - sinh^2 x = 1$
2. $tanh^2 x + sech^2 x = 1$
3. $coth^2 x - csch^2 x = 1$
4. $sinh^2 x = \frac{cosh2x - 1}{2}$
5. $sinh2x = 2sinhx \cdot coshx$
6. $sinh(x + y) = sinhx\,coshy + coshx\,sinhy$
7. $sinh(x - y) = sinhx\,coshy - coshx\,sinhy$
8. $cosh(x + y) = coshx\,coshy + sinhx\,sinhy$
8. $cosh(x - y) = coshx\,coshy - sinhx\,sinhy$
9. $cosh2x = cosh^2 x + sinh^2 x$

❒ Derivadas de las funciones hiperbólicas

1. $\frac{d[sinh\,u]}{dx} = coshu\,\frac{du}{dx}$
2. $\frac{d[cosh\,u]}{dx} = sinhu\,\frac{du}{dx}$
3. $\frac{d[tanh\,u]}{dx} = sech^2 u\,\frac{du}{dx}$
4. $\frac{d[coth\,u]}{dx} = -csch^2 u\,\frac{du}{dx}$
5. $\frac{d[sech\,u]}{dx} = -sechu \cdot tanhu\,\frac{du}{dx}$
6. $\frac{d[csch\,u]}{dx} = -cschu \cdot cothu\,\frac{du}{dx}$

❒ Integrales de funciones hiperbólica (Antiderivada)

1. $\int coshu\,du = sinhu + c$
2. $\int sinhu\,du = coshu + c$
3. $\int sech^2 u\,du = tanhu + c$
4. $\int csc^2 u\,du = -cothu + c$
5. $\int sechu \cdot tanhu\,du = -sechu + c$
6. $\int cschu \cdot coth\,du = -cschu + c$

❐ Funciones hiperbólicas inversas (definición)

1. $\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right) \qquad x \in \mathbb{R}$

2. $\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\right) \qquad x \in [1, \infty)$

3. $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad x \in (-1, 1)$

4. $\coth^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad x \in (-\infty, -1) \cup (1, \infty)$

5. $\operatorname{sech}^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \qquad x \in (0, 1]$

6. $\operatorname{csch}^{-1}x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right) \qquad x \in (-\infty, 0) \cup (0, \infty)$

❐ Derivadas de funciones hiperbólicas inversas

1. $\frac{d[\sinh^{-1}u]}{dx} = \frac{1}{\sqrt{u^2+1}}\ \frac{du}{dx}$

2. $\frac{d[\cosh^{-1}u]}{dx} = \frac{1}{\sqrt{u^2-1}}\ \frac{du}{dx}$

3. $\frac{d[\tanh^{-1}u]}{dx} = \frac{1}{1-u^2}\ \frac{du}{dx}$

4. $\frac{d[\coth^{-1}u]}{dx} = \frac{1}{1-u^2}\ \frac{du}{dx}$

5. $\frac{d[\operatorname{sech}^{-1}u]}{dx} = -\frac{1}{u\sqrt{1-u^2}}\ \frac{du}{dx}$

6. $\frac{d[\operatorname{csch}^{-1}u]}{dx} = -\frac{1}{|u|\sqrt{1+u^2}}\ \frac{du}{dx}$

❐ Integrales de funciones hiperbólica inversas

1. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + c$

2. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + c$

3. $\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 \pm u^2}}{|u|}\right) + c$