CLAVE

[1] $x^2y'' - 7xy' + 16y = 0$ con $y_1 = x^4$

$\Rightarrow y'' - \frac{7}{x}y' + \frac{16}{x^2}y = 0$

$\Rightarrow P(x) = -\frac{7}{x} \quad \Rightarrow y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2}dx$

$$\Rightarrow y_2 = x^4 \int \frac{e^{7\int \frac{dx}{x}}}{x^8}dx = x^4 \int \frac{e^{7\ln x}}{x^8}dx$$

$$= x^4 \int \frac{e^{\ln x^7}}{x^8}dx$$

$$= x^4 \int \frac{x^7}{x^8}dx = x^4 \int \frac{dx}{x}$$

$$= x^4 \ln x. \quad (20\text{pts})$$

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[2] $y'' + 4y' + 4y = 0$ que pase por los puntos $(0,2)$ y $(2,0)$

(20pts)

$\Rightarrow m^2 + 4m + 4 = 0$

$\Rightarrow (m+2)^2 = 0 \Rightarrow m_1 = m_2 = -2.$

$\Rightarrow y_h(x) = C_1 e^{-2x} + C_2 x e^{-2x}$, luego como pasa por $(2,0)$ y $(0,2)$

* Para $P(2,0)$

$$\Rightarrow 0 = C_1 e^{-4} + C_2 2 e^{-4} = e^{-4}(C_1 + 2C_2) = 0$$

$$\Rightarrow e^{-4} \neq 0$$

$$\Rightarrow C_1 + 2C_2 = 0$$

$$C_1 = -2C_2. \quad (1)$$

Para $(0,2)$

$\Rightarrow 2 = C_1 + 0 \Rightarrow C_1 = 2$

$\Rightarrow$ y por (1) $\Rightarrow C_1 = -2C_2$

$\Rightarrow 2 = -2C_2 \Rightarrow \boxed{C_2 = -1}$

entonces: $y_h(x) = 2e^{-2x} - xe^{-2x}$

$= e^{-2x}(2-x)$

③ $16y^{(IV)} + 24y'' + 9y = 0 \Rightarrow 16m^4 + 24m^2 + 9 = 0$

Sea $\alpha = m^2 \Rightarrow 16\alpha^2 + 24\alpha + 9 = 0$

$\Rightarrow \alpha = \dfrac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{32}$

$\alpha = \dfrac{-24 \pm \sqrt{576 - 576}}{32}$

$\alpha = -\dfrac{24}{32} = -\dfrac{3}{4}$

$\alpha_1 = -\frac{3}{4}$ y $\alpha_2 = -\frac{3}{4}$

$\Rightarrow m^2 = -\frac{3}{4} \Rightarrow m = \pm\sqrt{-\frac{3}{4}} = \pm \frac{1}{2}\sqrt{3}\,i$ de multiplicidad 2

$\Rightarrow y_h(x) = C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \operatorname{sen}\frac{\sqrt{3}}{2} + C_3 x \cos\frac{\sqrt{3}}{2}x + C_4 x \operatorname{sen}\frac{\sqrt{3}}{2}x.$

Otra forma de Resolver el problema (3)

$16y^{(IV)} + 24y'' + 9y = 0$

$\Rightarrow 16m^4 + 24m^2 + 9 = 0 \Rightarrow$ Factorizamos

$\Rightarrow 16m^4 + 24m^2 + 9 = 0$

$4m^2 \qquad 3 = 12m^2$

$4m^2 \qquad 3 = 12m^2$

$\qquad\qquad \overline{24m^2}$

$\Rightarrow (4m^2+3)(4m^2+3) = 0 \Rightarrow (4m^2+3)^2 = 0$

Tiene multiplicidad (2).

$\Rightarrow m = \pm\sqrt{-\frac{3}{4}} = \pm \frac{\sqrt{3}}{2} i \Rightarrow \alpha = 0 \wedge \beta = \frac{\sqrt{3}}{2}$

$\Rightarrow y_h(x) = C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \operatorname{sen} \frac{\sqrt{3}}{2} x + C_3 x \cos \frac{\sqrt{3}}{2} + C_4 x \operatorname{sen} \frac{\sqrt{3}}{2} x$

(5) $x^2 y'' + xy' + y = \cot(\ln x)$.

Sea $x = e^z \Rightarrow \ln x = z$, $x > 0$

y usamos el resultado para transformar

$$a_0 x^2 y'' + a_1 x y' + a_2 y = f(x)$$

a esta forma de coeficiente constantes, es decir.

$$\Rightarrow a_0 y''(z) + (a_1 - a_0) y'(z) + a_2 y = f(z).$$

Luego entonces: tenemos que:

$$\Rightarrow y''(z) + \cancel{(1-1)}^{\to 0} y'(z) + y(x) = \cot(z)$$

$$\Rightarrow y''(z) + y(x) = \cot(z).$$

$$\Rightarrow m^2 + 1 = 0 \Rightarrow m = \pm i$$

$$\Rightarrow y_h(z) = C_1 \cos z + C_2 \operatorname{sen} z.$$

$$W = \begin{vmatrix} \cos z & \operatorname{sen} z \\ -\operatorname{sen} z & \cos z \end{vmatrix} = \cos^2 z + \operatorname{sen}^2 z = 1.$$

$$W_1 = \begin{vmatrix} 0 & \operatorname{sen} z \\ \cot z & \cos z \end{vmatrix} = 0 - \cot z \operatorname{sen} z = -\left(\frac{\cos z}{\cancel{\operatorname{sen} z}}\right) \cancel{\operatorname{sen} z}$$

$$= -\cos z.$$

$$W_2 = \begin{vmatrix} \cos z & 0 \\ -\operatorname{sen} z & \cot z \end{vmatrix} = \cos z \left(\frac{\cos z}{\operatorname{sen} z}\right) = \frac{\cos^2 z}{\operatorname{sen} z}.$$

$$u_1 = \int \frac{W_1}{W}\,dz = -\int \cos z\,dz$$

$$= -\operatorname{Sen} z$$

$$u_2 = \int \frac{W_2}{W}\,dz = \int \frac{\cos^2 z}{\operatorname{sen} z}\,dz = \int \frac{1-\operatorname{Sen}^2 z}{\operatorname{sen} z}\,dz$$

$$u_2 = \int \frac{1}{\operatorname{sen} z}\,dz - \int \operatorname{sen} z\,dz$$

$$= \int \csc z\,dz - \int \operatorname{sen} z\,dz = \ln|\csc z - \cot z| + \cos z.$$

$$u_2 = \ln|\csc z - \cot z| + \cos z.$$

$$\Rightarrow y_p = u_1 y_1 + u_2 y_2 \Rightarrow y_p = \cancel{-\cos z \operatorname{sen} z} + \cancel{\operatorname{sen} z \cos z} + \ln|\csc z - \cot z|$$

$$y_p = \ln|\csc z - \cot z|$$

$$\Rightarrow y_G(z) = C_1 \cos z + C_2 \operatorname{sen} z. + \ln|\csc z - \cot z|$$

$$\Rightarrow y_G(x) = C_1 \cos \ln x + C_2 \operatorname{sen} \ln x + \ln|\csc \ln x - \cot x|$$

(20pts)

ⓔ $X'' + X = \cos \omega t \qquad X(0) = 0 \qquad X'(0) = 0$

$m^2 + 1 = 0 \Rightarrow m = \pm i$

$\Rightarrow X_h(t) = C_1 \cos t + C_2 \,\text{sen}\, t.$

$\Rightarrow$ Sea $X_p(t) = A \cos \omega t + B \,\text{sen}\, \omega t.$

$\Rightarrow X_p' = -A\omega \,\text{sen}\, \omega t + B\omega \cos \omega t.$

$X_p'' = -A\omega^2 \cos \omega t - B\omega^2 \,\text{sen}\, \omega t.$

$\Rightarrow$ Remplazamos

$-A\omega^2 \cos \omega t - B\omega^2 \,\text{sen}\,\omega t + A \cos \omega t + B \,\text{sen}\,\omega t = \cos \omega t.$

$\Rightarrow (-A\omega^2 + A) \cos \omega t + (-B\omega^2 + B) \,\text{sen}\,\omega t = \cos \omega t.$

$\Rightarrow (-A\omega^2 + A) = 1 \Rightarrow A(1-\omega^2) = 1$

$\Rightarrow \boxed{A = \frac{1}{1-\omega^2}}$

$(-B\omega^2 + B) = 0$

$\Rightarrow B(1-\omega^2) = 0 \Rightarrow \boxed{B = 0}$

$\Rightarrow X_p(t) = \frac{1}{1-\omega^2} \cos \omega t + 0$

$\Rightarrow X_G(t) = C_1 \cos t + C_2 \,\text{sen}\, t + \frac{1}{1-\omega^2} \cos \omega t.$

Usando la condiciones iniciales.

ⓐ $X(0) = 0$

$\Rightarrow 0 = C_1 + 0 + \frac{1}{1-\omega^2} \Rightarrow C_1 = -\frac{1}{1-\omega^2}.$

⇒ Para $x'(0) = 0$

Derivamos $X_4(t)$

$$\Rightarrow X_4'(t) = -C_1 \operatorname{sen} t + C_2 \cos t - \frac{w}{1-w^2} \operatorname{sen} wt.$$

⇒ Usamos $x'(0) = 0$

$$\Rightarrow 0 = -\cancel{C_1(0)} + C_2(1) - \cancel{\frac{w}{1-w^2}(0)}.$$

$$C_2 = 0$$

$$\Rightarrow X_4(t) = -\frac{1}{1-w^2} \cos t + \frac{1}{1-w^2} \cos wt.$$

$$X_4(t) = \frac{1}{1-w^2}\left(\cos wt - \cos t\right)$$

Finalmente para la posición cuando $w = 0.7$ rad y $t = 5$, tenemos:

$$\Rightarrow X_4(5) = \frac{1}{1-(0.7)^2}\left(\cos(0.7)(5) - \cos 5\right)$$

$$= \frac{1}{1-0.49}\left(-0.94 - 0.28\right)$$

$$= \frac{1}{0.51}\left(-1.22\right) = -2.39$$

≈ -2.39