Fórmulas de la Transformada de Laplace

Definición: $\pounds\{f(t)\} = \int_0^\infty e^{-st}f(t)\,dt = F(s)$

Propiedades:	Inversas:	
1. $\pounds\{1\} = \frac{1}{s}$	$\pounds^{-1}\{\frac{1}{s}\} = 1$	
2. $\pounds\{t^n\} = \frac{n!}{s^{n+1}}$	$\pounds^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$	$\to \pounds^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$
3. $\pounds\{e^{at}\} = \frac{1}{s-a}$	$\pounds^{-1}\{\frac{1}{s-a}\} = e^{at}$	
4. $\pounds\{\sin kt\} = \frac{k}{s^2+k^2}$	$\pounds^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin kt$	$\to \pounds^{-1}\left\{\frac{1}{s^2+k^2}\right\} = \frac{\sin kt}{k}$
5. $\pounds\{\cos kt\} = \frac{s}{s^2+k^2}$	$\pounds^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos kt$	
6. $\pounds\{\sinh kt\} = \frac{k}{s^2-k^2}$	$\pounds^{-1}\left\{\frac{k}{s^2-k^2}\right\} = \sinh kt$	$\to \pounds^{-1}\left\{\frac{1}{s^2-k^2}\right\} = \frac{\sinh kt}{k}$
7. $\pounds\{\cosh kt\} = \frac{s}{s^2-k^2}$	$\pounds^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh kt$	
8. $\pounds\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$	$\pounds^{-1}\left\{\frac{n!}{(s-a)^{n+1}}\right\} = t^n e^{at}$	$\to \pounds^{-1}\left\{\frac{1}{(s-a)^{n+1}}\right\} = \frac{t^n e^{at}}{n!}$

- **Teoremas**:

9. $\pounds\{e^{at}f(t)\} = \pounds\{f(t)\}_{s\to s-a} = F(s-a) \to \pounds^{-1}\{F(s-a)\} = e^{at}f(t)$ (1er Teorema de Traslación)

10. $\pounds\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\pounds\{f(t)\} = (-1)^n \frac{d^n}{ds^n}F(s) \to \pounds^{-1}\left\{(-1)^n \frac{d^n}{ds^n}\pounds\{f(t)\}\right\} = t^n f(t)$

En forma alternativa: $f(t) = \frac{(-1)^n}{t^n}\frac{d^n}{ds^n}F(s)$

Para $n = 1$ se tiene $f(t) = -\frac{1}{t}\frac{d}{ds}F(s)$ (Derivada de la Transformada)

11. $\pounds\{U(t-a)f(t-a)\} = e^{-as}\pounds\{f(t)\}$ (2do Teorema de Traslación)

La inversa: $\pounds^{-1}\{e^{-as}\pounds\{f(t)\}\} = U(t-a)f(t-a)$

- Forma alternativa: $\pounds\{U(t-a)g(t)\} = e^{-as}\pounds\{g(t+a)\}$

La inveresa: $\pounds^{-1}\{e^{-as}\pounds\{g(t+a)\}\} = U(t-a)g(t)$

12. $£\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$

$£\{y'(t)\} = sY(s) - y(0)$ y para $£\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$

(Transformada de la derivada)

12. $£\{f * g\} = £\{f(t)\} \bullet £\{g(t)\} = F(s) \bullet G(s)$ donde $(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$

(Transformada de una comvolución)

- $£^{-1}\{F(s) \bullet G(s)\} = f * g = \int_0^t f(\tau)g(t-\tau)d\tau$

13. $£\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}£\{f(t)\} = \frac{1}{s}F(s)$ **(Transformada de la Integral)**

- La inversa: $£^{-1}\{\frac{1}{s}F(s)\} = \int_0^t f(t)dt$

14. Si $f(t)$ es una función períodica con periodo T, entonces $£\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$