Forma General para transformar las Ecuaciones de Cauchy - Euler (orden 2) a Ecuaciones Dif. de orden dos con coeficientes constantes.

Sea

$$a_0 x^2 y''(x) + a_1 x y'(x) + a_2 y(x) = F(x) \qquad (1)$$

Usamos:

$$x = e^z \Rightarrow \ln x = z \quad , x > 0$$

$$\Rightarrow \frac{dx}{dz} = e^z \Rightarrow \frac{dz}{dx} = e^{-z} \quad (**)$$

$$\Rightarrow e^{-z} = \frac{dz}{dx} \cdot \frac{dy}{dy} = \frac{dy}{dx} \cdot \frac{dz}{dy}$$

$$\Rightarrow e^{-z} = y'(x) \frac{dz}{dy}$$

$$\Rightarrow y'(x) = e^{-z} \frac{dy}{dz} \quad (*) \Rightarrow y'(x) = e^{-z} y'(z)$$

Luego para

$$y''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx} y'(x) \cdot \frac{\frac{1}{dz}}{\frac{1}{dz}} \quad (**)$$

$$y''(x) = \frac{d y'(x)}{dz} \cdot \frac{dz}{dx} = \frac{d}{dz}\left(e^{-z} \frac{dy}{dz}\right) \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(e^{-z} \cdot y'(z)\right) e^{-z}$$

$$y''(x) = \cancel{A} \Rightarrow \left(y''(z) e^{-z} - y'(z) e^{-z}\right) e^{-z}$$

$$\Rightarrow e^{-2z}\left(y''(z) - y'(z)\right)$$

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Por lo tanto tenemos:

$$y'(x) = e^{-z} y'(z)$$

$$y''(x) = e^{-2z}\left(y''(z) - y'(z)\right)$$

Ahora Sustituyendo: en la ecuación (1)

$$\Rightarrow a_0 x^2 \left[e^{-2z}\left(y''(z) - y'(z)\right)\right] + a_1 x\left[e^{-z} y'(z)\right] + a_2 y(x) = F(x)$$

Y como $x = e^z$ y $x^2 = e^{2z}$

$\Rightarrow$

$$\Rightarrow a_0 (e^{2z})\left[e^{-2z}\left(y''(z) - y'(z)\right)\right] + a_1 e^z\left[e^{-z} y'(z)\right] + a_2 y(x) = F(x)$$

$$\Rightarrow a_0\left(y''(z) - y'(z)\right) + a_1 y'(z) + a_2 y(x) = F(x)$$

$$\Rightarrow a_0 y''(z) - a_0 y'(z) + a_1 y'(z) + a_2 y(x) = F(x)$$

$$\Rightarrow a_0 y''(z) + (a_1 - a_0) y'(z) + a_2 y(x) = F(x) \quad (*)$$

Ejemplo: Resolver: $x^2 y'' + x y' + y = \cot(\ln x)$

- Sea $x = e^z \Rightarrow \ln x = z$, $x > 0$

$$\Rightarrow y''(z) + (1-1) y'(z) + y(x) = \cot(\ln x)$$

$$\Rightarrow y''(z) + y(x) = \cot z$$

$$\Rightarrow m^2 + 1 = 0 \Rightarrow m = \pm i$$

$$\Rightarrow y_h(z) = C_1 \operatorname{Sen} z + C_2 \operatorname{Cos} z$$

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Para $y_p(z) = u_1 y_1 + u_2 y_2$ (3)

$$W = \begin{vmatrix} \operatorname{sen} z & \cos z \\ \cos z & -\operatorname{sen} z \end{vmatrix} = -\operatorname{sen}^2 z - \cos z = -1$$

$$W_1 = \begin{vmatrix} 0 & \cos z \\ \cot z & -\operatorname{sen} z \end{vmatrix} = 0 - \cos z \cot z = -\frac{\cos^2 z}{\operatorname{sen} z}$$

$$W_2 = \begin{vmatrix} \operatorname{sen} z & 0 \\ \cos z & \cot z \end{vmatrix} = \operatorname{sen} z \cot z = \cos z$$

$$\Rightarrow u_1 = -\int \frac{\cos^2 z}{\operatorname{sen} z} dz = \int \frac{\operatorname{sen}^2 z - 1}{\operatorname{sen} z} dz = \int \operatorname{sen} z \, dz - \int \csc z \, dz$$

$$\Rightarrow -\cos z + \ln|\csc z - \cot z| = u_1$$

$$y \quad u_2 = \int \cos z \, dz = \operatorname{sen} z$$

$$\Rightarrow u_1 y_1 = \ln|\csc z - \cot z| . \operatorname{sen} z - \cos z \operatorname{sen} z$$

$$u_2 y_2 = \operatorname{sen} z \cos z .$$

$$\Rightarrow y_p(z) = \ln|\csc z - \cot z| \operatorname{sen} z - \cancel{\cos z \operatorname{sen} z} + \cancel{\cos z \operatorname{sen} z}$$

$$y_p(z) = \ln|\csc z - \cot z| \operatorname{sen} z$$

$$y_G(z) = C_1 \operatorname{sen} z + C_2 \cos z + \ln|\csc z - \cot z| \operatorname{sen} z$$

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Finalmente para $X = e^{z} \Rightarrow z = \ln|x|$ , $X > 0$

$$\Rightarrow y_{p}(x) = C_1 \operatorname{sen}(\ln x) + C_2 \cos(\ln x) + \ln|\csc(\ln x) - \cot(\ln x)| \operatorname{sen}(\ln x)$$

Fin//

* Resuelva las siguientes Ecuaciones usando el método anterior

[1] $x^2 y'' + 5xy' + 4y = 16x^2 - 8\ln x$

[2] $3x^2 y'' + 11xy' - 3y = 5 + 4x^{-3}$

[3] $2x^2 y'' + y = 5 \operatorname{sen}(\ln x)$

[4] $x^2 y'' + x y' + y = \tan(\ln x)$