7) $2\sqrt{x}\,y' - y = -\sin\sqrt{x} - \cos\sqrt{x}$

$\to \frac{dy}{dx} - \frac{1}{2\sqrt{x}}y = -\frac{1}{2\sqrt{x}}(\sin\sqrt{x} + \cos\sqrt{x})$ lineal

$\to P(x) = -\frac{1}{2\sqrt{x}} \quad ; f(x) = -\frac{1}{2\sqrt{x}}(\sin\sqrt{x} + \cos\sqrt{x})$

$$\to ye^{-\frac{1}{2}\int\frac{dx}{\sqrt{x}}} = -\frac{1}{2}\int\frac{(\sin\sqrt{x}+\cos\sqrt{x})}{\sqrt{x}}e^{-\frac{1}{2}\int\frac{dx}{\sqrt{x}}}dx + c$$

$$\to ye^{-\sqrt{x}} = -\frac{1}{2}\int\frac{(\sin\sqrt{x}+\cos\sqrt{x})}{\sqrt{x}}e^{-\sqrt{x}}dx + c$$

$$\to ye^{-\sqrt{x}} = -\frac{1}{2}\int\frac{\sin\sqrt{x}}{\sqrt{x}}e^{-\sqrt{x}}dx - \frac{1}{2}\int\frac{\cos\sqrt{x}}{\sqrt{x}}e^{-\sqrt{x}}dx + c$$

$\to$ Hacemos $k = \sqrt{x} \Rightarrow dk = \frac{1}{2\sqrt{x}}dx$

$$\to ye^{-\sqrt{x}} = -\int e^{-k}\sin k\,dk - \int e^{-k}\cos k\,dk + c$$

$$\to ye^{-\sqrt{x}} = -[-\tfrac{1}{2}e^{-k}\cos k - \tfrac{1}{2}e^{-k}\sin k] - [\tfrac{1}{2}e^{-k}\sin k - \tfrac{1}{2}e^{-k}\cos k] + c$$

$$\to ye^{-\sqrt{x}} = \tfrac{1}{2}e^{-k}\cos k + \tfrac{1}{2}e^{-k}\sin k - \tfrac{1}{2}e^{-k}\sin k + \tfrac{1}{2}e^{-k}\cos k = e^{-k}\cos k + c$$

$$\to ye^{-\sqrt{x}} = e^{-k}\cos k + c$$

$$\Rightarrow ye^{-\sqrt{x}} = e^{-\sqrt{x}}\cos\sqrt{x} + c$$

$\Rightarrow y = \cos\sqrt{x} + ce^{-\sqrt{x}}$, si $x \to \infty$ entonces $y = \cos\sqrt{x}$

$*\int e^{-k}\sin k\,dk = -\frac{1}{2}e^{-k}\cos k - \frac{1}{2}e^{-k}\sin k$

observación sobre la integración

$**\int e^{-k}\cos k\,dk = \frac{1}{2}e^{-k}\sin k - \frac{1}{2}e^{k}\cos k$

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8) $2\sin x y' + y\cos x = y^3(x\cos x - \sin x)$

dividimos por $2\sin x$

$\to y' + \frac{1}{2}y\cot x = \frac{1}{2}y^3(x\cot x - 1)$ Bernoullí donde: $P(x) = \frac{1}{2}\cot x$; $n = 3$; $f(x) = \frac{1}{2}(x\cot x - 1)$

luego usamos $y^{(1-n)}e^{(1-n)\int P(x)dx} = (1-n)\int f(x)e^{(1-n)\int P(x)dx}dx + c$

$\rightarrow y^{-2}e^{-2\int \frac{1}{2}\cot x dx} = -2\int \frac{1}{2}(x\cot x - 1)e^{-2\int \frac{1}{2}\cot x dx}dx + c$

$\rightarrow y^{-2}e^{-\ln \sin x} = -\int (x\cot x - 1)e^{-\ln \sin x}dx + c$

$\rightarrow y^{-2}\frac{1}{\sin x} = -\int (x\cot x - 1)\frac{1}{\sin x}dx + c$

$\rightarrow y^{-2}\frac{1}{\sin x} = -\int x\frac{\cos x}{\sin^2 x}dx + \int \frac{1}{\sin x}dx + c$

Integrando por parte la primera integral.

$u = x \qquad dv = \frac{\cos x}{\sin^2 x}dx$

$du = dx \qquad v = \int \frac{\cos x}{\sin^2 x}dx = -\frac{1}{\sin x}$

remplazamos en la expresión

$\rightarrow y^{-2}\frac{1}{\sin x} = -\left(-\frac{x}{\sin x} + \int \frac{1}{\sin x}dx\right) + \int \frac{1}{\sin x}dx + c$

$\rightarrow y^{-2}\frac{1}{\sin x} = \frac{x}{\sin x} - \int \frac{1}{\sin x}dx + \int \frac{1}{\sin x}dx + c$

$\Rightarrow y^{-2}\frac{1}{\sin x} = \frac{x}{\sin x} + c$

$\Rightarrow \frac{1}{y^2} = x + c\sin x$