1. $(y + x\cot\frac{y}{x})dx - xdy = 0$

$M(1,u) = u + \cot u$

$N(1,u) = -1$

$\Rightarrow \ln x + \int \frac{-1}{u+\cot u-u} du = c$

$\Rightarrow \ln x - \int \frac{du}{\cot u} = c$

$\Rightarrow \ln x - \int \tan u du = c$

$\Rightarrow \ln x - (-\ln(\cos u)) = \ln x + \ln(\cos u) = c$

$\Rightarrow \ln(x\cos u) = c$

$\Rightarrow x\cos\frac{y}{x} = c_1$

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2 $\left(x + \sqrt{y^2 - xy}\right)\frac{dy}{dx} = y$

$\Rightarrow -ydx + \left(x + \sqrt{y^2 - xy}\right)dy = 0$

$x = vy \Rightarrow v = \frac{x}{y}$

$M(v,1) = -1$
$N(v,1) = v + \sqrt{1 - v}$

$\Rightarrow \ln y + \int \frac{-1}{v+\sqrt{1-v}-v} dv = c$

$\Rightarrow \ln y - \int \frac{dv}{\sqrt{1-v}} = c$

$\int \frac{dv}{\sqrt{1-v}} = -2\sqrt{1 - v}$

$\Rightarrow \ln y + 2\sqrt{1 - v} = c$