

**FORMULARIO DE  
CÁLCULO DIFERENCIAL  
E INTEGRAL**

VER.3.6

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**VALOR ABSOLUTO**

$$|a| = \begin{cases} a & \text{si } a \geq 0 \\ -a & \text{si } a < 0 \end{cases}$$

$$|a| = |-a|$$

$$a \leq |a| \quad y \quad -a \leq |a|$$

$$|a| \geq 0 \quad y \quad |a| = 0 \Leftrightarrow a = 0$$

$$|ab| = |a||b| \quad \text{ó} \quad \left| \prod_{k=1}^n a_k \right| = \prod_{k=1}^n |a_k|$$

$$|a+b| \leq |a|+|b| \quad \text{ó} \quad \left| \sum_{k=1}^n a_k \right| \leq \sum_{k=1}^n |a_k|$$

**EXPONENTES**

$$a^p \cdot a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$(a \cdot b)^p = a^p \cdot b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$a^{p/q} = \sqrt[q]{a^p}$$

**LOGARITMOS**

$$\log_a N = x \Rightarrow a^x = N$$

$$\log_a MN = \log_a M + \log_a N$$

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

$$\log_a N^r = r \log_a N$$

$$\log_a N = \frac{\log_b N}{\log_b a} = \frac{\ln N}{\ln a}$$

$$\log_{10} N = \log N \quad y \quad \log_e N = \ln N$$

**ALGUNOS PRODUCTOS**

$$a \cdot (c+d) = ac+ad$$

$$(a+b) \cdot (a-b) = a^2 - b^2$$

$$(a+b) \cdot (a+b) = (a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b) \cdot (a-b) = (a-b)^2 = a^2 - 2ab + b^2$$

$$(x+b) \cdot (x+d) = x^2 + (b+d)x + bd$$

$$(ax+b) \cdot (cx+d) = acx^2 + (ad+bc)x + bd$$

$$(a+b) \cdot (c+d) = ac+ad+bc+bd$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$(a-b) \cdot (a^2 + ab + b^2) = a^3 - b^3$$

$$(a-b) \cdot (a^3 + a^2b + ab^2 + b^3) = a^4 - b^4$$

$$(a-b) \cdot (a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$$

$$(a-b) \cdot \left(\sum_{k=1}^n a^{n-k} b^{k-1}\right) = a^n - b^n \quad \forall n \in \mathbb{N}$$

$$(a+b) \cdot (a^2 - ab + b^2) = a^3 + b^3$$

$$(a+b) \cdot (a^3 - a^2b + ab^2 - b^3) = a^4 - b^4$$

$$(a+b) \cdot (a^4 - a^3b + a^2b^2 - ab^3 + b^4) = a^5 + b^5$$

$$(a+b) \cdot (a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5) = a^6 - b^6$$

$$(a+b) \cdot \left(\sum_{k=1}^n (-1)^{k+1} a^{n-k} b^{k-1}\right) = a^n + b^n \quad \forall n \in \mathbb{N} \text{ impar}$$

$$(a+b) \cdot \left(\sum_{k=1}^n (-1)^{k+1} a^{n-k} b^{k-1}\right) = a^n - b^n \quad \forall n \in \mathbb{N} \text{ par}$$

**SUMAS Y PRODUCTOS**

$$a_1 + a_2 + \dots + a_n = \sum_{k=1}^n a_k$$

$$\sum_{k=1}^n c = nc$$

$$\sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k$$

$$\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

$$\sum_{k=1}^n (a_k - a_{k-1}) = a_n - a_0$$

$$\sum_{k=1}^n [a + (k-1)d] = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}(a+l)$$

$$\sum_{k=1}^n ar^{k-1} = a \frac{1-r^n}{1-r} = \frac{a-r l}{1-r}$$

$$\sum_{k=1}^n k = \frac{1}{2}(n^2 + n)$$

$$\sum_{k=1}^n k^2 = \frac{1}{6}(2n^3 + 3n^2 + n)$$

$$\sum_{k=1}^n k^3 = \frac{1}{4}(n^4 + 2n^3 + n^2)$$

$$\sum_{k=1}^n k^4 = \frac{1}{30}(6n^5 + 15n^4 + 10n^3 - n)$$

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

$$n! = \prod_{k=1}^n k$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}, \quad k \leq n$$

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

$$(x_1 + x_2 + \dots + x_n)^n = \sum \frac{n!}{n_1! n_2! \dots n_k!} x_1^{n_1} \cdot x_2^{n_2} \cdot \dots \cdot x_k^{n_k}$$

**CONSTANTES**

$$\pi = 3.14159265359\dots$$

$$e = 2.71828182846\dots$$

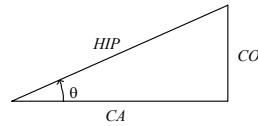
**TRIGONOMETRÍA**

$$\text{sen } \theta = \frac{CO}{HIP} \quad \text{csc } \theta = \frac{1}{\text{sen } \theta}$$

$$\text{cos } \theta = \frac{CA}{HIP} \quad \text{sec } \theta = \frac{1}{\text{cos } \theta}$$

$$\text{tg } \theta = \frac{\text{sen } \theta}{\text{cos } \theta} = \frac{CO}{CA} \quad \text{ctg } \theta = \frac{1}{\text{tg } \theta}$$

$$\pi \text{ radianes} = 180^\circ$$



$\theta$	sen	cos	tg	ctg	sec	csc
$0^\circ$	0	1	0	$\infty$	1	$\infty$
$30^\circ$	1/2	$\sqrt{3}/2$	$1/\sqrt{3}$	$\sqrt{3}$	$2/\sqrt{3}$	2
$45^\circ$	$1/\sqrt{2}$	$1/\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60^\circ$	$\sqrt{3}/2$	1/2	$\sqrt{3}$	$1/\sqrt{3}$	2	$2/\sqrt{3}$
$90^\circ$	1	0	$\infty$	0	$\infty$	1

$$y = \angle \text{sen } x \quad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$y = \angle \text{cos } x \quad y \in [0, \pi]$$

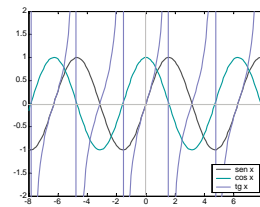
$$y = \angle \text{tg } x \quad y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$y = \angle \text{ctg } x = \angle \text{tg } \frac{1}{x} \quad y \in (0, \pi)$$

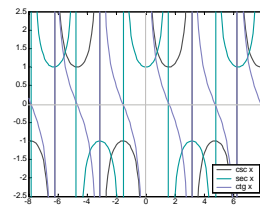
$$y = \angle \text{sec } x = \angle \text{cos } \frac{1}{x} \quad y \in [0, \pi]$$

$$y = \angle \text{csc } x = \angle \text{sen } \frac{1}{x} \quad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

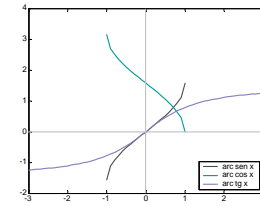
Gráfica 1. Las funciones trigonométricas:  $\text{sen } x$ ,  $\text{cos } x$ ,  $\text{tg } x$ :



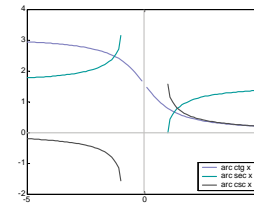
Gráfica 2. Las funciones trigonométricas  $\text{csc } x$ ,  $\text{sec } x$ ,  $\text{ctg } x$ :



Gráfica 3. Las funciones trigonométricas inversas  $\text{arcsen } x$ ,  $\text{arccos } x$ ,  $\text{arctg } x$ :



Gráfica 4. Las funciones trigonométricas inversas  $\text{arctg } x$ ,  $\text{arcsec } x$ ,  $\text{arccsc } x$ :



**IDENTIDADES TRIGONOMÉTRICAS**

$$\text{sen}^2 \theta + \text{cos}^2 \theta = 1$$

$$1 + \text{ctg}^2 \theta = \text{csc}^2 \theta$$

$$\text{tg}^2 \theta + 1 = \text{sec}^2 \theta$$

$$\text{sen}(-\theta) = -\text{sen } \theta$$

$$\text{cos}(-\theta) = \text{cos } \theta$$

$$\text{tg}(-\theta) = -\text{tg } \theta$$

$$\text{sen}(\theta + 2\pi) = \text{sen } \theta$$

$$\text{cos}(\theta + 2\pi) = \text{cos } \theta$$

$$\text{tg}(\theta + 2\pi) = \text{tg } \theta$$

$$\text{sen}(\theta + \pi) = -\text{sen } \theta$$

$$\text{cos}(\theta + \pi) = -\text{cos } \theta$$

$$\text{tg}(\theta + \pi) = \text{tg } \theta$$

$$\text{sen}(\theta + m\pi) = (-1)^m \text{sen } \theta$$

$$\text{cos}(\theta + m\pi) = (-1)^m \text{cos } \theta$$

$$\text{tg}(\theta + m\pi) = \text{tg } \theta$$

$$\text{sen}(m\pi) = 0$$

$$\text{cos}(m\pi) = (-1)^m$$

$$\text{tg}(m\pi) = 0$$

$$\text{sen}\left(\frac{2n+1}{2}\pi\right) = (-1)^n$$

$$\text{cos}\left(\frac{2n+1}{2}\pi\right) = 0$$

$$\text{tg}\left(\frac{2n+1}{2}\pi\right) = \infty$$

$$\text{sen } \theta = \text{cos}\left(\theta - \frac{\pi}{2}\right)$$

$$\text{cos } \theta = \text{sen}\left(\theta + \frac{\pi}{2}\right)$$

$$\text{sen}(\alpha \pm \beta) = \text{sen } \alpha \text{ cos } \beta \pm \text{cos } \alpha \text{ sen } \beta$$

$$\text{cos}(\alpha \pm \beta) = \text{cos } \alpha \text{ cos } \beta \mp \text{sen } \alpha \text{ sen } \beta$$

$$\text{tg}(\alpha \pm \beta) = \frac{\text{tg } \alpha \pm \text{tg } \beta}{1 \mp \text{tg } \alpha \text{ tg } \beta}$$

$$\text{sen } 2\theta = 2 \text{sen } \theta \text{ cos } \theta$$

$$\text{cos } 2\theta = \text{cos}^2 \theta - \text{sen}^2 \theta$$

$$\text{tg } 2\theta = \frac{2 \text{tg } \theta}{1 - \text{tg}^2 \theta}$$

$$\text{sen}^2 \theta = \frac{1}{2}(1 - \text{cos } 2\theta)$$

$$\text{cos}^2 \theta = \frac{1}{2}(1 + \text{cos } 2\theta)$$

$$\text{tg}^2 \theta = \frac{1 - \text{cos } 2\theta}{1 + \text{cos } 2\theta}$$

$$\text{sen } \alpha + \text{sen } \beta = 2 \text{sen} \frac{1}{2}(\alpha + \beta) \cdot \text{cos} \frac{1}{2}(\alpha - \beta)$$

$$\text{sen } \alpha - \text{sen } \beta = 2 \text{sen} \frac{1}{2}(\alpha - \beta) \cdot \text{cos} \frac{1}{2}(\alpha + \beta)$$

$$\text{cos } \alpha + \text{cos } \beta = 2 \text{cos} \frac{1}{2}(\alpha + \beta) \cdot \text{cos} \frac{1}{2}(\alpha - \beta)$$

$$\text{cos } \alpha - \text{cos } \beta = -2 \text{sen} \frac{1}{2}(\alpha + \beta) \cdot \text{sen} \frac{1}{2}(\alpha - \beta)$$

$$\text{tg } \alpha \pm \text{tg } \beta = \frac{\text{sen}(\alpha \pm \beta)}{\text{cos } \alpha \cdot \text{cos } \beta}$$

$$\text{sen } \alpha \cdot \text{cos } \beta = \frac{1}{2}[\text{sen}(\alpha - \beta) + \text{sen}(\alpha + \beta)]$$

$$\text{sen } \alpha \cdot \text{sen } \beta = \frac{1}{2}[\text{cos}(\alpha - \beta) - \text{cos}(\alpha + \beta)]$$

$$\text{cos } \alpha \cdot \text{cos } \beta = \frac{1}{2}[\text{cos}(\alpha - \beta) + \text{cos}(\alpha + \beta)]$$

$$\text{tg } \alpha \cdot \text{tg } \beta = \frac{\text{tg } \alpha + \text{tg } \beta}{\text{ctg } \alpha + \text{ctg } \beta}$$

**FUNCIONES HIPERBÓLICAS**

$$\text{senh } x = \frac{e^x - e^{-x}}{2}$$

$$\text{cosh } x = \frac{e^x + e^{-x}}{2}$$

$$\text{tgh } x = \frac{\text{senh } x}{\text{cosh } x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{ctgh } x = \frac{1}{\text{tgh } x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\text{sech } x = \frac{1}{\text{cosh } x} = \frac{2}{e^x + e^{-x}}$$

$$\text{csch } x = \frac{1}{\text{senh } x} = \frac{2}{e^x - e^{-x}}$$

$$\text{senh} : \mathbb{R} \rightarrow \mathbb{R}$$

$$\text{cosh} : \mathbb{R} \rightarrow [1, \infty)$$

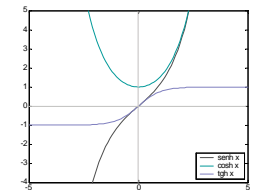
$$\text{tgh} : \mathbb{R} \rightarrow \langle -1, 1 \rangle$$

$$\text{ctgh} : \mathbb{R} - \{0\} \rightarrow \langle -\infty, -1 \rangle \cup \langle 1, \infty \rangle$$

$$\text{sech} : \mathbb{R} \rightarrow \langle 0, 1 \rangle$$

$$\text{csch} : \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{0\}$$

Gráfica 5. Las funciones hiperbólicas  $\text{senh } x$ ,  $\text{cosh } x$ ,  $\text{tgh } x$ :



**FUNCIONES HIPERBÓLICAS INVERSAS**

$$\text{senh}^{-1} x = \ln(x + \sqrt{x^2 + 1}), \quad \forall x \in \mathbb{R}$$

$$\text{cosh}^{-1} x = \ln(x + \sqrt{x^2 - 1}), \quad x \geq 1$$

$$\text{tgh}^{-1} x = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|, \quad |x| < 1$$

$$\text{ctgh}^{-1} x = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right|, \quad |x| > 1$$

$$\text{sech}^{-1} x = \ln \left( \frac{1 + \sqrt{1-x^2}}{x} \right), \quad 0 < x \leq 1$$

$$\text{csch}^{-1} x = \ln \left( \frac{1 + \sqrt{x^2 + 1}}{|x|} \right), \quad x \neq 0$$

## IDENTIDADES DE FUNCS HIP

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \operatorname{tgh}^2 x = \operatorname{sech}^2 x$$

$$\operatorname{ctgh}^2 x - 1 = \operatorname{csch} x$$

$$\sinh(-x) = -\sinh x$$

$$\cosh(-x) = \cosh x$$

$$\operatorname{tgh}(-x) = -\operatorname{tgh} x$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\operatorname{tgh}(x \pm y) = \frac{\operatorname{tgh} x \pm \operatorname{tgh} y}{1 \pm \operatorname{tgh} x \operatorname{tgh} y}$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\operatorname{tgh} 2x = \frac{2 \operatorname{tgh} x}{1 + \operatorname{tgh}^2 x}$$

$$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$$

$$\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$$

$$\operatorname{tgh}^2 x = \frac{\cosh 2x - 1}{\cosh 2x + 1}$$

$$\operatorname{tgh} x = \frac{\sinh 2x}{\cosh 2x + 1}$$

## OTRAS

$$ax^2 + bx + c = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = \text{discriminante}$$

## LÍMITES

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e = 2.71828\dots$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\ln x} = 1$$

## DERIVADAS

$$D_x f(x) = \frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(cx) = c$$

$$\frac{d}{dx}(cx^n) = ncx^{n-1}$$

$$\frac{d}{dx}(u \pm v \pm w \pm \dots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \dots$$

$$\frac{d}{dx}(cu) = c \frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx}$$

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

$$\frac{dF}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} \quad (\text{Regla de la Cadena})$$

$$\frac{du}{dx} = \frac{1}{dx/du}$$

$$\frac{dF}{dx} = \frac{dF/du}{dx/du}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{f_2'(t)}{f_1'(t)} \quad \text{donde} \begin{cases} x = f_1(t) \\ y = f_2(t) \end{cases}$$

## DERIVADA DE FUNCS LOG & EXP

$$\frac{d}{dx}(\ln u) = \frac{du/dx}{u} = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\log u) = \frac{\log e}{u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\log_a u) = \frac{\log_a e}{u} \cdot \frac{du}{dx} \quad a > 0, a \neq 1$$

$$\frac{d}{dx}(e^x) = e^x \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(a^x) = a^x \ln a \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(u^v) = vu^{v-1} \frac{du}{dx} + \ln u \cdot u^v \cdot \frac{dv}{dx}$$

## DERIVADA DE FUNCIONES TRIGO

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{tg} u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{ctg} u) = -\operatorname{csc}^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \operatorname{tg} u \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{csc} u) = -\operatorname{csc} u \operatorname{ctg} u \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{vers} u) = \sin u \frac{du}{dx}$$

## DERIV DE FUNCS TRIGO INVER

$$\frac{d}{dx}(\angle \sin u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\angle \cos u) = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\angle \operatorname{tg} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\angle \operatorname{ctg} u) = -\frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\angle \sec u) = \pm \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx} \quad \begin{cases} + \text{ si } u > 1 \\ - \text{ si } u < -1 \end{cases}$$

$$\frac{d}{dx}(\angle \operatorname{csc} u) = \mp \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx} \quad \begin{cases} - \text{ si } u > 1 \\ + \text{ si } u < -1 \end{cases}$$

$$\frac{d}{dx}(\angle \operatorname{vers} u) = \frac{1}{\sqrt{2u-u^2}} \cdot \frac{du}{dx}$$

## DERIVADA DE FUNCS HIPERBÓLICAS

$$\frac{d}{dx} \sinh u = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx} \cosh u = \sinh u \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{tgh} u = \operatorname{sech}^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{ctgh} u = -\operatorname{csch}^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{sech} u = -\operatorname{sech} u \operatorname{tgh} u \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{csch} u = -\operatorname{csch} u \operatorname{ctgh} u \frac{du}{dx}$$

## DERIVADA DE FUNCS HIP INV

$$\frac{d}{dx} \sinh^{-1} u = \frac{1}{\sqrt{1+u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \cosh^{-1} u = \frac{\pm 1}{\sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad u > 1 \begin{cases} + \text{ si } \cosh^{-1} u > 0 \\ - \text{ si } \cosh^{-1} u < 0 \end{cases}$$

$$\frac{d}{dx} \operatorname{tgh}^{-1} u = \frac{1}{1-u^2} \cdot \frac{du}{dx}, \quad |u| < 1$$

$$\frac{d}{dx} \operatorname{ctgh}^{-1} u = \frac{1}{1-u^2} \cdot \frac{du}{dx}, \quad |u| > 1$$

$$\frac{d}{dx} \operatorname{sech}^{-1} u = \frac{\mp 1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx} \quad \begin{cases} - \text{ si } \operatorname{sech}^{-1} u > 0, u \in (0,1) \\ + \text{ si } \operatorname{sech}^{-1} u < 0, u \in (0,1) \end{cases}$$

$$\frac{d}{dx} \operatorname{csch}^{-1} u = -\frac{1}{|u|\sqrt{1+u^2}} \cdot \frac{du}{dx}, \quad u \neq 0$$

## INTEGRALES DEFINIDAS, PROPIEDADES

$$\int_a^b \{f(x) \pm g(x)\} dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^b cf(x) dx = c \cdot \int_a^b f(x) dx \quad c \in \mathbb{R}$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$\int_a^b f(x) dx = -\int_b^a f(x) dx$$

$$\int_a^a f(x) dx = 0$$

$$m \cdot (b-a) \leq \int_a^b f(x) dx \leq M \cdot (b-a)$$

$$\Leftrightarrow m \leq f(x) \leq M \quad \forall x \in [a,b], m, M \in \mathbb{R}$$

$$\int_a^b f(x) dx \leq \int_a^b g(x) dx$$

$$\Leftrightarrow f(x) \leq g(x) \quad \forall x \in [a,b]$$

$$\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx \quad \text{si } a < b$$

## INTEGRALES

$$\int adx = ax$$

$$\int af(x) dx = a \int f(x) dx$$

$$\int (u \pm v \pm w \pm \dots) dx = \int u dx \pm \int v dx \pm \int w dx \pm \dots$$

$$\int u dv = uv - \int v du \quad (\text{Integración por partes})$$

$$\int u^n du = \frac{u^{n+1}}{n+1} \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u|$$

## INTEGRALES DE FUNCS LOG & EXP

$$\int e^x dx = e^x$$

$$\int a^x dx = \frac{a^x}{\ln a} \quad \begin{cases} a > 0 \\ a \neq 1 \end{cases}$$

$$\int ua^x dx = \frac{a^x}{\ln a} \cdot \left( u - \frac{1}{\ln a} \right)$$

$$\int ue^x dx = e^x(u-1)$$

$$\int \ln u du = u \ln u - u + u(\ln u - 1)$$

$$\int \log_a u du = \frac{1}{\ln a} (u \ln u - u) = \frac{u}{\ln a} (\ln u - 1)$$

$$\int u \log_a u du = \frac{u^2}{4} \cdot (2 \log_a u - 1)$$

$$\int u \ln u du = \frac{u^2}{4} (2 \ln u - 1)$$

## INTEGRALES DE FUNCS TRIGO

$$\int \sin u du = -\cos u$$

$$\int \cos u du = \sin u$$

$$\int \sec^2 u du = \operatorname{tg} u$$

$$\int \operatorname{csc}^2 u du = -\operatorname{ctg} u$$

$$\int \sec u \operatorname{tg} u du = \sec u$$

$$\int \operatorname{csc} u \operatorname{ctg} u du = -\operatorname{csc} u$$

$$\int \operatorname{tg} u du = -\ln|\cos u| = \ln|\sec u|$$

$$\int \operatorname{ctg} u du = \ln|\sin u|$$

$$\int \sec u du = \ln|\sec u + \operatorname{tg} u|$$

$$\int \operatorname{csc} u du = \ln|\operatorname{csc} u - \operatorname{ctg} u|$$

$$\int \sec^2 u du = \frac{u}{2} - \frac{1}{4} \sin 2u$$

$$\int \cos^2 u du = \frac{u}{2} + \frac{1}{4} \sin 2u$$

$$\int \operatorname{tg}^2 u du = \operatorname{tg} u - u$$

$$\int \operatorname{ctg}^2 u du = -(\operatorname{ctg} u + u)$$

$$\int u \sin u du = \sin u - u \cos u$$

$$\int u \cos u du = \cos u + u \sin u$$

## INTEGRALES DE FUNCS TRIGO INV

$$\int \angle \sin u du = u \angle \sin u + \sqrt{1-u^2}$$

$$\int \angle \cos u du = u \angle \cos u - \sqrt{1-u^2}$$

$$\int \angle \operatorname{tg} u du = u \angle \operatorname{tg} u - \ln|\sqrt{1+u^2}|$$

$$\int \angle \operatorname{ctg} u du = u \angle \operatorname{ctg} u + \ln|\sqrt{1+u^2}|$$

$$\int \angle \sec u du = u \angle \sec u - \ln(u + \sqrt{u^2-1})$$

$$\int \angle \operatorname{csc} u du = u \angle \operatorname{csc} u + \ln(u + \sqrt{u^2-1})$$

## INTEGRALES DE FUNCS HIP

$$\int \sinh u du = \cosh u$$

$$\int \cosh u du = \sinh u$$

$$\int \operatorname{sech}^2 u du = \operatorname{tgh} u$$

$$\int \operatorname{csch}^2 u du = -\operatorname{ctgh} u$$

$$\int \operatorname{sech} u \operatorname{tgh} u du = -\operatorname{sech} u$$

$$\int \operatorname{csch} u \operatorname{ctgh} u du = -\operatorname{csch} u$$

$$\int \operatorname{tgh} u du = \ln \cosh u$$

$$\int \operatorname{ctgh} u du = \ln|\sinh u|$$

$$\int \operatorname{sech} u du = \angle \operatorname{tg}(\sinh u)$$

$$\int \operatorname{csch} u du = -\operatorname{ctgh}^{-1}(\cosh u)$$

$$= \ln \operatorname{tgh} \frac{1}{2} u$$

## INTEGRALES DE FRAC

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \angle \operatorname{tg} \frac{u}{a}$$

$$= -\frac{1}{a} \angle \operatorname{ctg} \frac{u}{a}$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \frac{u-a}{u+a} \quad (u^2 > a^2)$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \frac{a+u}{a-u} \quad (u^2 < a^2)$$

## INTEGRALES CON \sqrt

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \angle \sin \frac{u}{a}$$

$$= -\angle \cos \frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2})$$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = \frac{1}{a} \ln \left| \frac{u}{a + \sqrt{a^2 \pm u^2}} \right|$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \angle \operatorname{csc} \frac{u}{a}$$

$$= \frac{1}{a} \angle \operatorname{sec} \frac{u}{a}$$

$$\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \angle \sin \frac{u}{a}$$

$$\int \sqrt{u^2 \pm a^2} du = \frac{u}{2} \sqrt{u^2 \pm a^2} + \frac{a^2}{2} \ln(u + \sqrt{u^2 \pm a^2})$$

## MAS INTEGRALES

$$\int e^{mu} \sin bu du = \frac{e^{mu} (a \sin bu - b \cos bu)}{a^2 + b^2}$$

$$\int e^{mu} \cos bu du = \frac{e^{mu} (a \cos bu + b \sin bu)}{a^2 + b^2}$$

## ALGUNAS SERIES

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!}$$

$$+ \dots + \frac{f^{(n)}(x_0)(x-x_0)^n}{n!} : \text{Taylor}$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!}$$

$$+ \dots + \frac{f^{(n)}(0)x^n}{n!} : \text{Maclaurin}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^{n-1} \frac{x^{2n-2}}{(2n-2)!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n}$$

$$\angle \operatorname{tg} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$$